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INSTITUCIÓ CATALANA DE RECERCA I ESTUDIS AVANÇATS

Diffusion versus dispersal wave-of-advance models and Neolithic spread

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1. reaction-diffusion *vs* reaction-dispersal

Fisher's equation

$$\frac{\partial F}{\partial t} = D_F \nabla^2 F + a_F F \left(1 - \frac{F}{K_F}\right)$$

$F = F(x,y,t)$ = population density (e.g., farmers)

D_F = diffusion coefficient

Logistic growth:

a_F = initial growth rate

K_F = carrying capacity

speed of range expansions = $2\sqrt{a_F D_F}$

Fisher's equation is only an approximation*, obtained from:

$$F(x,y,t+T) - F(x,y,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F\left(x+\Delta_x, y+\Delta_y, t\right)\ \phi_F\left(\Delta_x,\Delta_y\right) d\Delta_x\, d\Delta_y - F(x,y,t) + R_T[F(x,y,t)] - F(x,y,t)$$

Logistic

*Fort & Méndez, Phys. Rev. Lett. (1999)

We will compare the speed from this equation to Fisher's approximation = $2\sqrt{a_F D_F}$

Preindustrial populations (farmers)

Population A*: $\{p_j\}$={0.54, 0.17, 0.04, 0.25},

$\{r_j\}$={2.4, 14.5, 36.3, 60.4}km.

Population B*: $\{p_j\}$={0.40, 0.17, 0.17, 0.26},

$\{r_j\}$={2.4, 14.5, 36.3, 60.4}km.

Population C*: $\{p_j\}$={0.19, 0.07, 0.22, 0.52},

$\{r_j\}$={2.4, 14.5, 36.2, 60.4}km.

Population D**: $\{p_j\}$={0.19, 0.54, 0.17, 0.04, 0.04, 0.02},

$\{r_j\}$={5, 30, 50, 70, 90, 110}km.

Population E***: $\{p_j\}$={0.42; 0.23; 0.16; 0.08; 0.07; 0.02; 0.01; 0.01},

$\{r_j\}$={2.3, 7.3, 15, 25, 35, 45, 55, 100}km.

*Ethiopia; **Brazil; ***Central African Republic

Preindustrial populations (farmers)

Values of a_F and T:

$0.023\ y^{-1} \leq a_F \leq 0.033\ y^{-1}$ (from 4 ethnographic and 1 archaeological populations)

$T = 32\ y$ (from ethnographic data)

Population	speed (km/yr)	Fisher (km/yr)	error Fisher
A	0.71-0.81	0.85-1.02	20%-26%
B	0.75-0.84	0.93-1.11	24%-32%
C	0.92-1.01	1.26-1.51	37%-**50%**
D	0.93-1.06	1.11-1.34	19%-26%
E	0.61-0.74	0.54-0.65	-11%--12%

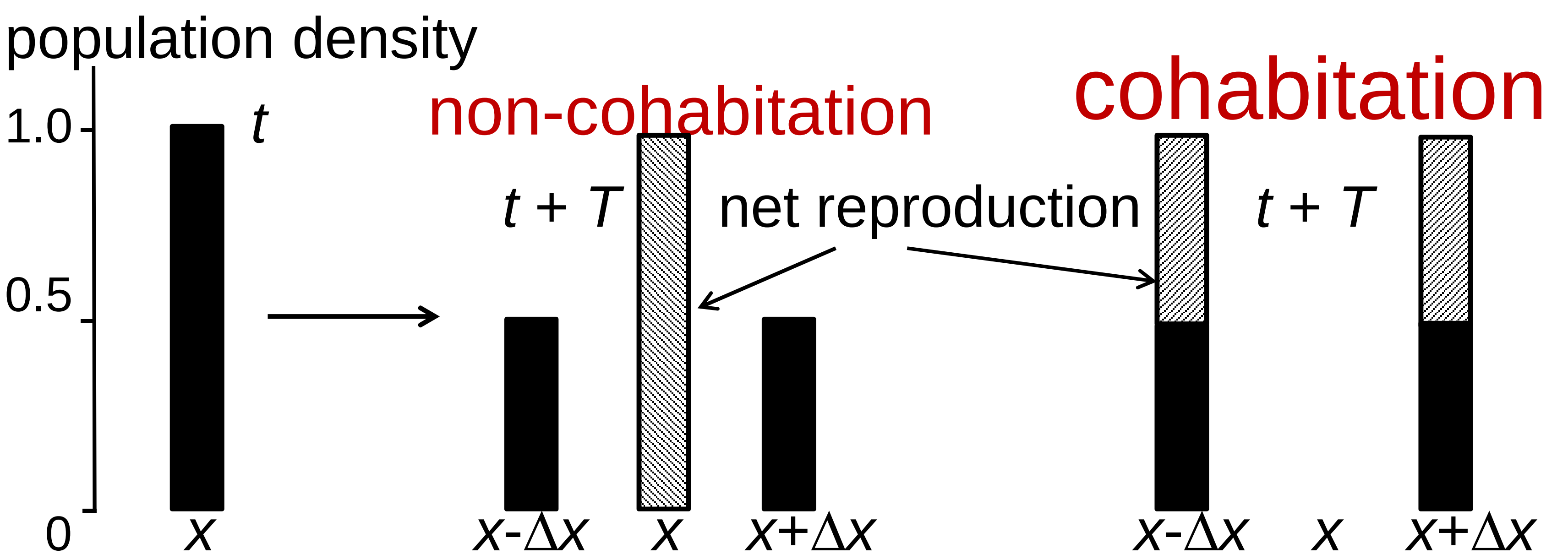
Isern, Fort & Pérez-Losada, JSTAT (2008)

2. non-cohabitation *vs* cohabitation eqs.

Up to now: non-cohabitation eq.: $F(x,y,t+T)-F(x,y,t)=$
$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(x+\Delta_x,y+\Delta_y,t)\ \phi_F(\Delta_x,\Delta_y)\,d\Delta_x\,d\Delta_y-F(x,y,t)+$
$R_T[F(x,y,t)]-F(x,y,t)$

Cohabitation equation:

$$F(x,y,t+T)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R_T\left[F\left(x+\Delta_x,y+\Delta_y,t\right)\right]\ \phi_F\left(\Delta_x,\Delta_y\right)d\Delta_x\,d\Delta_y$$



Preindustrial populations (farmers)

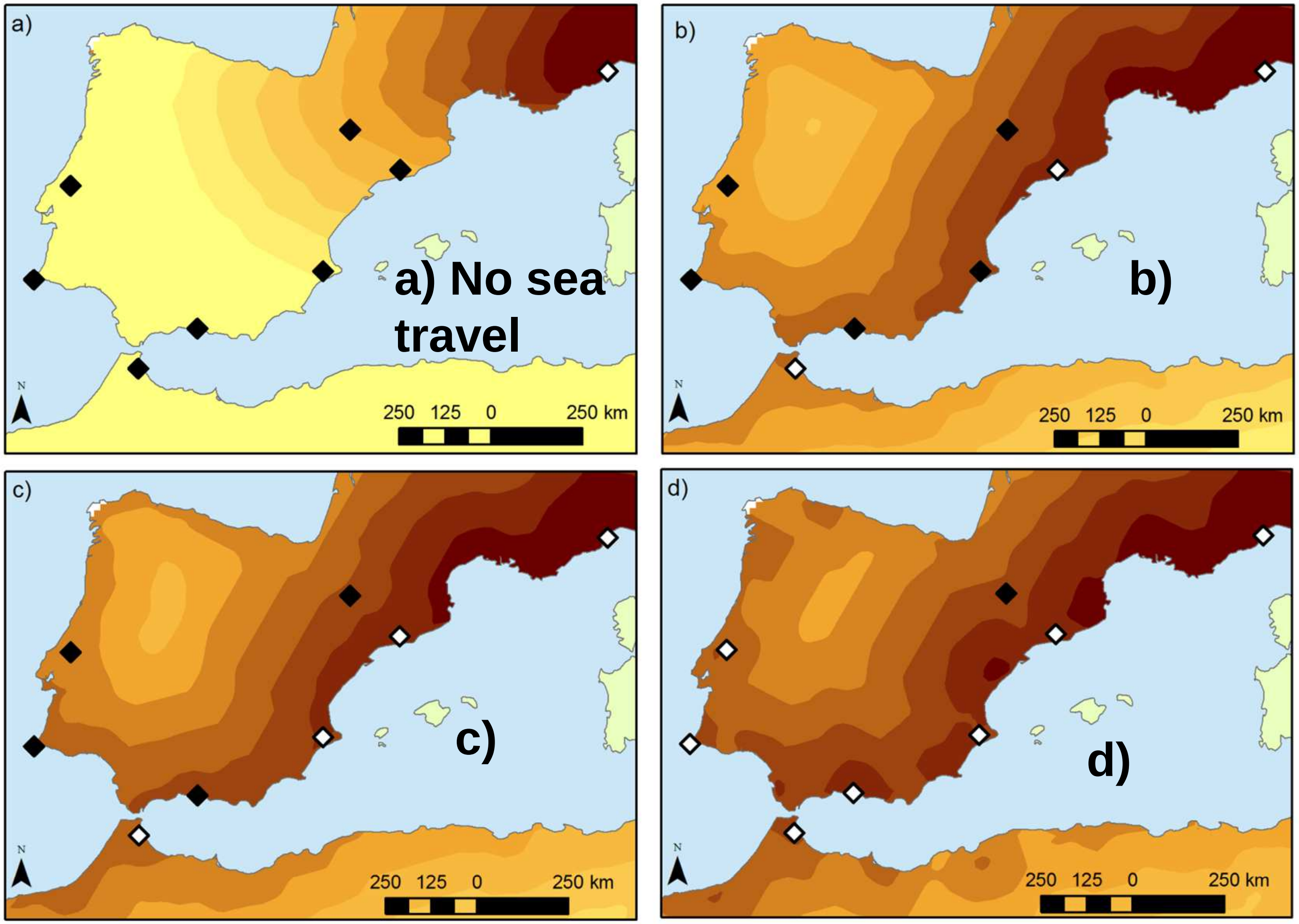
Population	Cohabitation (km/yr)	**error non-cohab. (relative to cohab.)**	error Fisher (relative to cohab.)
A	0.91-1.10	-22%--26%	-6%--8%
B	0.96-1.15	-22%--27%	-5%--3%
C	1.20-1.40	-23%--**28%**	5%-8%
D	1.18-1.44	-21%--26%	-6%--7%
E	0.74-0.94	-18%--22%	27-**31%**

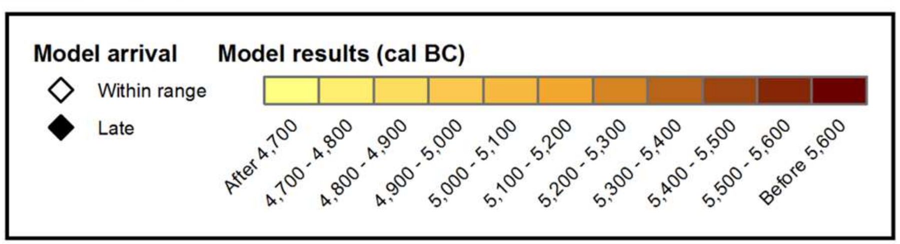
Another way to see the limitations of Fishers' eq.:

Fisher's speed = $2\sqrt{a_F D_F} \rightarrow \infty$ if $a_F \rightarrow \infty$

Cohabitation speed* $\rightarrow \frac{r_{max}}{T}$ if $a_F \rightarrow \infty$

$$\text{* cohabitation speed} = \min_{\lambda > 0} \frac{a_F\, T + \ln\left[\sum_{j=1}^{M} p_j\, I_0(\lambda r_j)\right]}{T\lambda}$$





b)-d) Sea travel up to 350 km

b) preference for closer destinations

b) all distances within 350 km equally likely

b) all jumps of 350 km

◇ within range

◆ too late

Isern, Zilhao, Fort & Ammerman, *PNAS* 2017

3. With cultural transmission (*F* = farmers, *H* =HGs)

$$\begin{cases} F(x,y,t+T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{F}(x+\Delta_x, y+\Delta_y, t)\, \phi_F\left(\Delta_x,\Delta_y\right) d\Delta_x\, d\Delta_y \\ H(x,y,t+T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{H}(x+\Delta_x, y+\Delta_y, t)\, \phi_H\left(\Delta_x,\Delta_y\right) d\Delta_x\, d\Delta_y \end{cases}$$

$$\tilde{F}(x,y,t) \equiv R_T[F(x,y,t)] + f\,\frac{R_T[F(x,y,t)]R_T[H(x,y,t)]}{R_T[F(x,y,t)] +\ \gamma\, R_T[H(x,y,t)]}$$

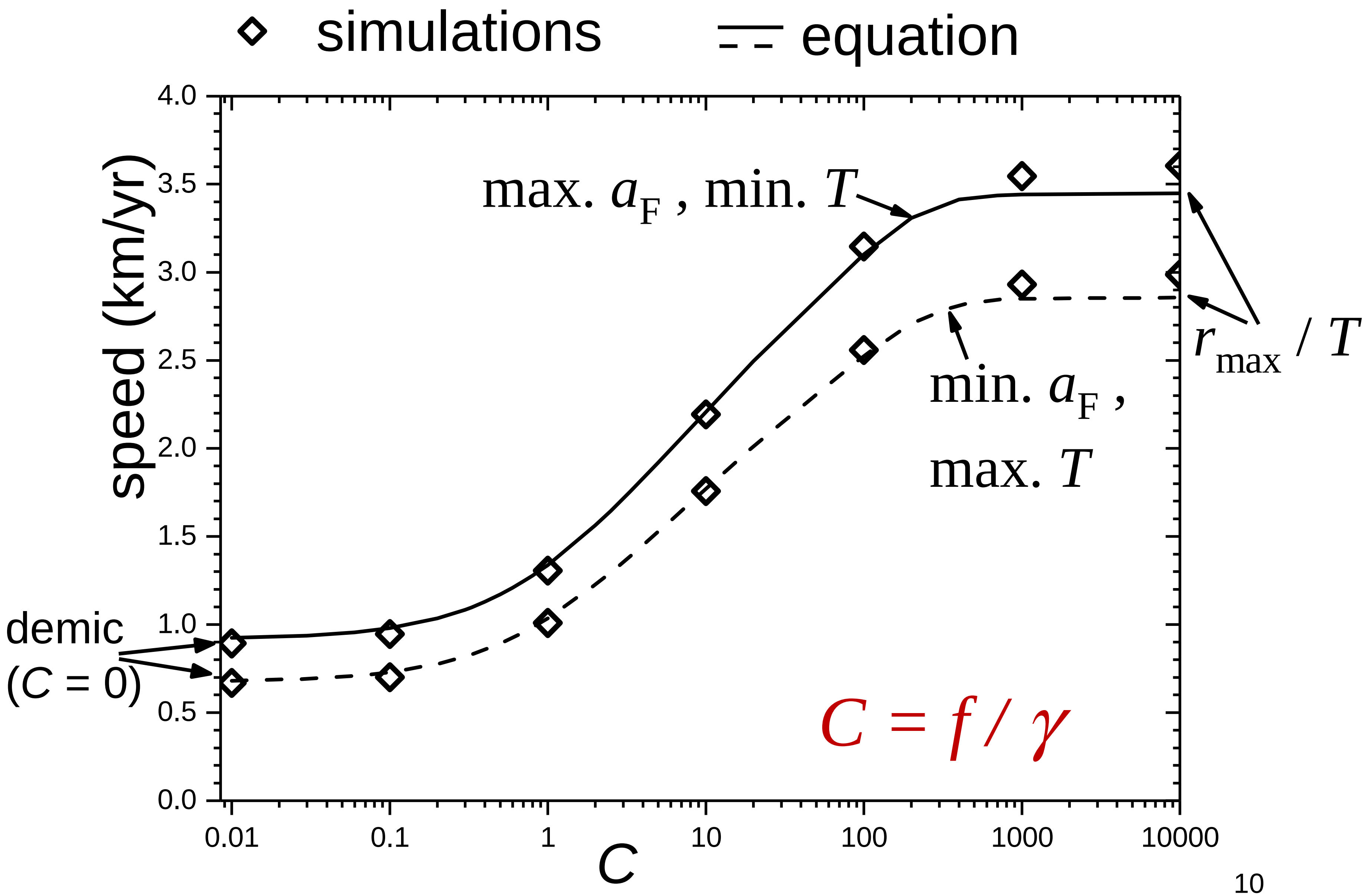
$$\tilde{H}(x,y,t) \equiv R_T[H(x,y,t)] - f\,\frac{R_T[F(x,y,t)]R_T[H(x,y,t)]}{R_T[F(x,y,t)] +\ \gamma\, R_T[H(x,y,t)]}$$

$$R_T[F(x,y,t)] = \frac{e^{a_F T}\, K_F\, F(x,y,t)}{K_F + (e^{a_F T} - 1)\, F(x,y,t)}$$

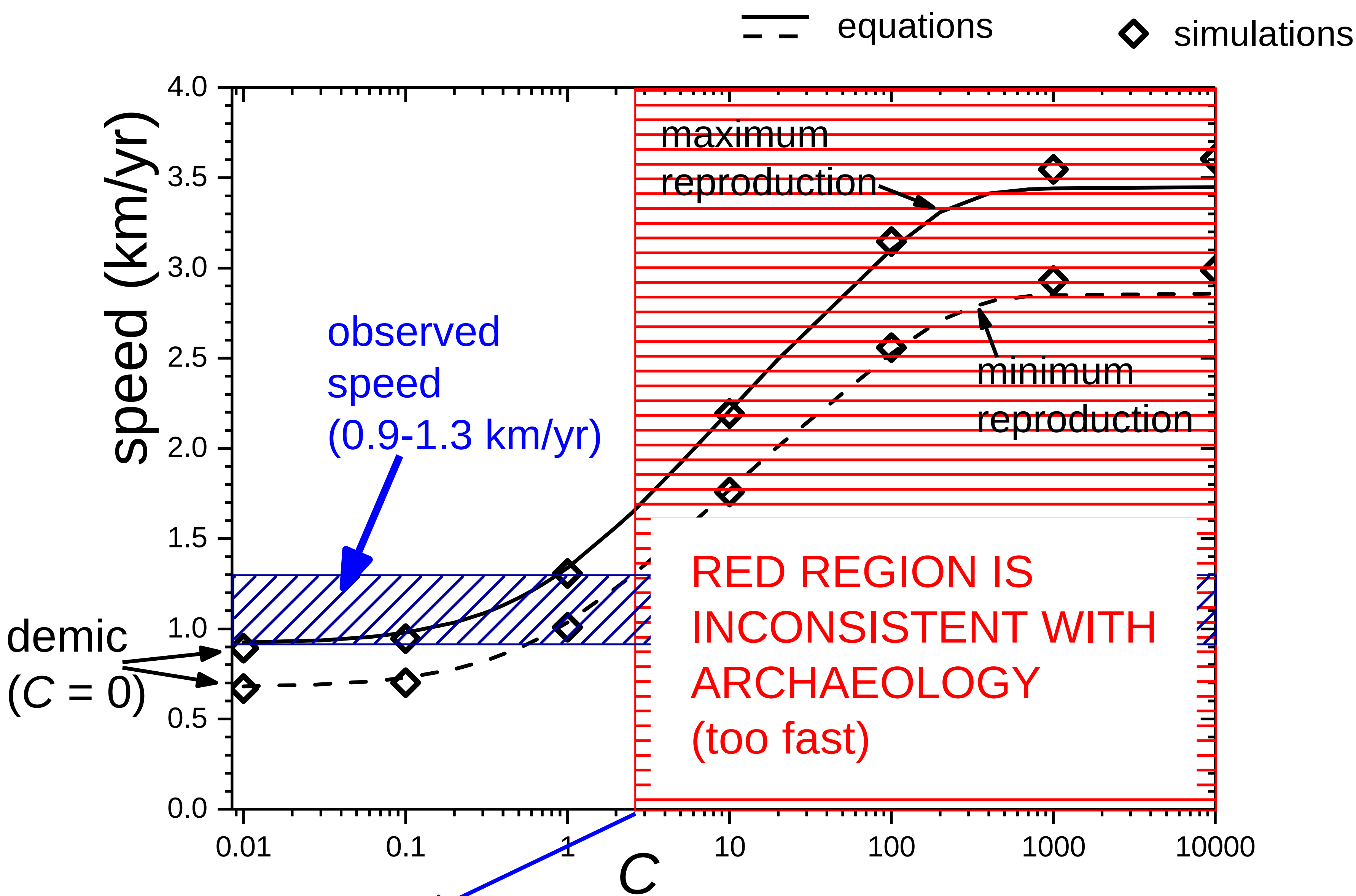
$$R_T[H(x,y,t)] = \frac{e^{a_H T}\, K_H\, H(x,y,t)}{K_H + \left(e^{a_H T} - 1\right) H(x,y,t)}$$

Fort, *PNAS* 2012

Plot of the front speed using the Eqs. in the previous slide



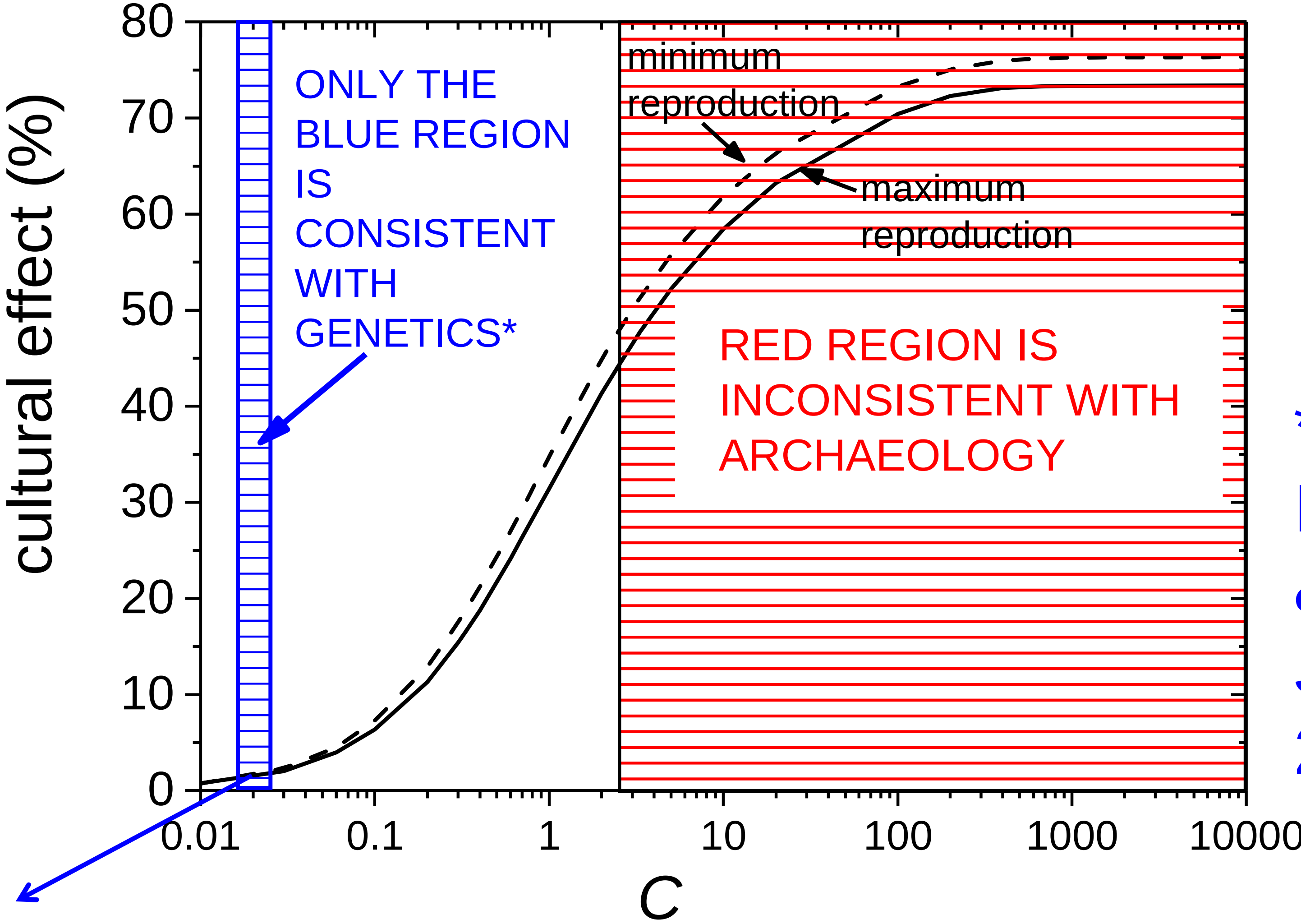
Fort, *PNAS* (2012)



$C < 3$, so less than 3 HGs were converted per F per generation

Fort, *PNAS* (2012)

Effect (%) = (speed – demic speed) /speed · 100



*Genetics: Isern, Fort & Rioja, *Sci. Rep.* 2017

cultural effect of only 2%, so demic>>cultural