

Do global string loops collapse to form black holes?

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Hawking has shown that the emission of gravitational radiation cannot prevent circular loops of gauged cosmic strings from collapsing into black holes. Here we consider the corresponding question for global strings: can Goldstone boson emission prevent circular loops of global cosmic strings from forming black holes? Our results show that for every value of the string tension there is a certain critical size below which the circular loop does not collapse to form a black hole. For GUT scale strings, this critical size is much larger than the current horizon.

Some years ago, Hawking [1] proved that a circular loop of gauge cosmic string would eventually collapse to form a black hole. Furthermore, he showed that in the process of collapse, the loop would radiate at most 29% of its total energy in gravitational radiation before forming a black hole. The analysis Hawking gave used the singularity theorems of gravity and did not depend on the field theoretic details of the gauge string.

In the present paper we will consider a circular loop of *global* string. Here the loop primarily radiates Goldstone bosons during its collapse and the question arises if a black hole can eventually form. It is unfortunate that there are no corresponding singularity theorems that can give information about non-gravitational radiation and so we have to use a method which is much less elegant than that used by Hawking: we explicitly find the energy lost in Goldstone bosons as a function of time and check if the loop ever collapses to within its own Schwarzschild radius. If it does, then a black hole will form while if the loop never falls within its Schwarzschild radius then a black hole will not form.

In order to find the energy lost by the loop into Goldstone boson emission, we have had to make a number of simplifying assumptions. For example, we have ignored the radiation back-reaction [2,3], the self-gravity of the loop and the energy lost to gravitational radiation. Because of these assumptions, our results cannot be considered rigorous. However, we feel that a substantially more complicated calculation without these simplifications would yield qualitatively similar results.

The simplest field theoretic action that gives global strings is

$$S = \int d^4x \left[\frac{1}{2} |\partial_\mu \phi|^2 - \frac{1}{4} \lambda (|\phi|^2 - \eta^2)^2 \right], \quad (1)$$

where ϕ is a complex scalar field. The dynamics of global string loops also follows from (1) but it seems that this can only be done numerically [4,3]. Instead a somewhat different approach is usually taken and the Kalb–Ramond action is considered [5,6]:

$$S = \frac{1}{6} \int F_{\mu\nu\sigma} F^{\mu\nu\sigma} + 2\pi\eta \int A_{\mu\nu} d\sigma^{\mu\nu} - \mu_0 \int d^2\sigma, \quad (2)$$

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where $A_{\mu\nu}$ is an antisymmetric tensor field,

$$F_{\mu\nu\sigma} = \partial_\mu A_{\nu\sigma} + \partial_\nu A_{\sigma\mu} + \partial_\sigma A_{\mu\nu}, \quad (3)$$

and the surface element of the string world sheet $x^\mu(\zeta, \tau)$ is

$$d\sigma^{\mu\nu} = c^{\mu\nu}(\zeta, \tau) d\zeta d\tau. \quad (4)$$

Here, ζ and τ parametrize the world-sheet and

$$c^{\mu\nu} = \dot{x}^\mu x'^\nu - \dot{x}^\nu x'^\mu. \quad (5)$$

Overdots and primes denote differentiation with respect to the time coordinate and ζ respectively. In addition,

$$d\sigma = (-\frac{1}{2}d\sigma_{\mu\nu}d\sigma^{\mu\nu})^{1/2}. \quad (6)$$

The connection between (1) and (2) is established by the relation [6]

$$\frac{1}{6}\epsilon_{\mu\nu\sigma\tau}F^{\nu\sigma\tau} = \eta\partial_\mu\theta, \quad (7)$$

where θ is the phase of the complex scalar field ϕ . One can also attempt to derive [4] (2) from (1) under suitable assumptions. Our attitude in the present work will be to simply adopt (2) as our starting point. This point of view is fully justified in the context of cosmic superstrings [5] as they are based precisely on the action in (2).

We now use the gauge choice

$$\partial_\nu A^{\mu\nu} = 0, \quad \dot{x}\cdot x' = 0, \quad \dot{x}^2 + x'^2 = 0, \quad \tau = t. \quad (8)$$

Then the equations of motion following from (2) are

$$\partial_\sigma\partial^\sigma A^{\mu\nu} = 4\pi j^{\mu\nu}, \quad (9)$$

$$j^{\mu\nu} = \frac{1}{2}\eta \int d\zeta \delta^3(\mathbf{x} - \mathbf{x}(\zeta, t)) c^{\mu\nu}(\zeta, t), \quad (10)$$

$$\mu_0(\ddot{x}_\mu - x''_\mu) = 4\pi\eta F_{\mu\nu\sigma}\dot{x}^\nu x'^\sigma. \quad (11)$$

The right-hand side of (11) gives the back reaction of the radiation on the dynamics of the string. It can be shown [7,2] that it also contains a term that renormalizes the bare string tension $\mu_0 \approx \eta^2$. If we ignore the radiation back reaction, the string dynamics is simply that of a Nambu-Goto string. For a circular loop of radius $R(t)$, the solution is

$$R(t) = R_0 \cos(t/R_0), \quad (12)$$

where R_0 is the radius of the loop at time $t=0$.

We now turn to the radiation from the circular loop. For this we must find the solution to (9) as a function of time. This is easily done by standard methods [8] and after using (10) we find

$$A^{\mu\nu}(\mathbf{x}, t) = \eta \int_0^{2\pi} d\zeta \int d\tau c^{\mu\nu}(\zeta, \tau) \Theta(t-\tau) \delta([\mathbf{x} - \mathbf{x}(\zeta, \tau)]^2). \quad (13)$$

For a given value of ζ , the integrand over τ will be non-zero only when τ equals the retarded time, t_r , which is defined by

$$t_r = t - |\mathbf{x} - \mathbf{x}(\zeta, t_r)|. \quad (14)$$

Differentiating eq. (13) and then using (3) gives a very lengthy expression for the field strength. However,

we are only interested in the radiation part of the field strength. This means that we should let $r = |\mathbf{x}| \rightarrow \infty$ and keep only the leading $1/r$ terms of the field strength. This procedure yields

$$F_{\text{rad}}^{\mu\nu\sigma} = \frac{\eta}{2r} \int d\zeta \left(\frac{\dot{c}^{\nu\sigma} n^\mu + \dot{c}^{\sigma\mu} n^\nu + \dot{c}^{\mu\nu} n^\sigma}{[n^\lambda \dot{x}_\lambda]^2} - \frac{c^{\nu\sigma} n^\mu + c^{\sigma\mu} n^\nu + c^{\mu\nu} n^\sigma}{[n^\lambda \dot{x}_\lambda]^3 n^\gamma \ddot{x}_\gamma} \right)_{t=t_r}, \quad (15)$$

where

$$n^\mu = \frac{x^\mu - x^\mu(\zeta, t_r)}{|\mathbf{x} - \mathbf{x}(\zeta, t_r)|} \quad (16)$$

is a null vector.

We are interested in the flux of energy radiated from the string. This can be found from the energy-momentum tensor [6]

$$T_{\text{rad}}^{0i} = -F_{i\alpha\beta}^{(\text{rad})} F_{(\text{rad})}^{0\alpha\beta} \quad (i \neq 0). \quad (17)$$

The energy radiated from the string is given by an integral of the energy flux over a sphere of radius r (which is taken to ∞):

$$\dot{E} = r^2 \int_0^\pi d\theta \int_0^{2\pi} d\phi \sin \theta e_i T_{(\text{rad})}^{0i}, \quad (18)$$

where e_i is the unit radial three vector

$$e_i = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta). \quad (19)$$

Putting together eqs. (15)–(19), we get

$$\dot{E} = \frac{\eta^2}{2} \int_0^\pi d\theta \sin \theta \cos^2 \theta \int_0^{2\pi} d\phi \left[\int_0^{2\pi} d\sigma \left(\frac{\dot{R}^2 + R\ddot{R}}{[1 - \dot{R} \sin \theta \cos(\sigma - \phi)]^2} + \frac{R\dot{R}\ddot{R} \sin \theta \cos(\sigma - \phi)}{[1 - \dot{R} \sin \theta \cos(\sigma - \phi)]^3} \right) \right]^2, \quad (20)$$

where R and its time derivatives are evaluated at the retarded time and $\sigma = \zeta/R_0$. Note that the retarded time in the radiation zone ($r \rightarrow \infty$) is given by $t_r = t - r$. Therefore, the effect of having the retarded time in (20) is simply to shift t and this shift may be absorbed by redefining t . The overall effect is equivalent to evaluating the integrand in (20) at time t and not at the retarded time t_r .

The integrations over σ and ϕ can now be done to yield

$$\dot{E} = \eta^2 \pi^3 \int_0^\pi d\theta \sin \theta \cos^2 \theta \left(\frac{2\dot{R}^2 + 2R\ddot{R}}{(1 - \dot{R}^2 \sin^2 \theta)^{3/2}} + \frac{3R\dot{R}\ddot{R} \sin^2 \theta}{(1 - \dot{R}^2 \sin^2 \theta)^{5/2}} \right)^2. \quad (21)$$

The integration over θ can be done by transforming the variable of integration to $u = \cos \theta$:

$$\dot{E}(t) = \eta^2 \pi^3 \left(\frac{88x + 16x \cos 4x + 12 \sin 4x - 19 \sin 8x}{512 \sin^3 x \cos^3 x} \right), \quad (22)$$

where $x = t/R_0$.

Next we need to find $E(t)$. For this we need to integrate (22) over t_r . We have done this integration numerically and the result is shown in fig. 1.

Our criterion for black hole formation is

$$\frac{2GM(t)}{R(t)} \geq 1, \quad (23)$$

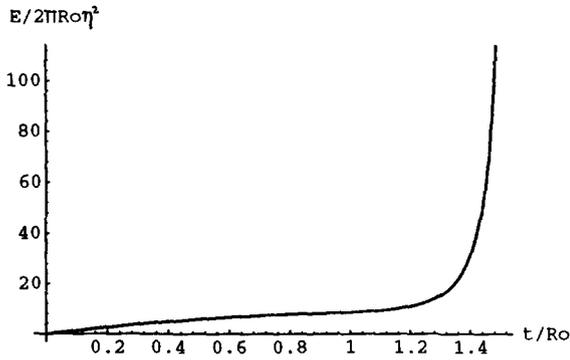


Fig. 1. The energy (per unit length) radiated from the loop, $E/2\pi R_0$, in units of η^2 as a function of time, t/R_0 .

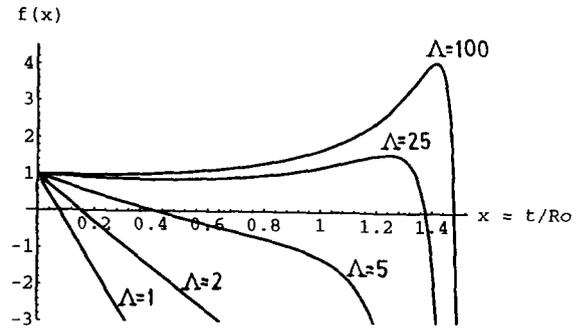


Fig. 2. The function $f(t)$ versus t/R_0 for various values of the parameter Λ .

for any time t . Here $M(t)$ is the energy of the loop at time t , that is

$$M(t) = 2\pi R_0 \mu - E(t). \tag{24}$$

Note that the initial energy, $2\pi R_0 \mu$, is given in terms of the *renormalized* string tension

$$\mu \approx \mu_0 \ln(\eta R_0) \equiv \eta^2 \Lambda, \tag{25}$$

where Λ varies logarithmically with R_0 .

Let us define a function $f(t)$ via

$$4\pi G \mu f(t) = \frac{2GM(t)}{R(t)}. \tag{26}$$

Using (12), (24), (25) and (26) we find

$$f(t) = \frac{1}{\cos(t/R_0)} \left(1 - \frac{E(t)}{2\pi R_0 \eta^2 \Lambda} \right). \tag{27}$$

The criterion for black hole formation now is $4\pi G \mu > 1/f(t)$ for some t .

In fig. 2 we plot $f(t)$ versus $x = t/R_0$ for values of Λ between 1 and 100. The behaviour of the plots is easily understandable in terms of two effects present in eq. (27): (i) the factor of $\cos(t/R_0)$ in the denominator – that is, the collapse of the loop – which tends to increase $f(t)$, and, (ii) the term $E(t)$ – that is, the energy lost to radiation – which tends to decrease $f(t)$. For small values of Λ , the effect of the radiation is very strong and the decrease in $f(t)$ due to the rapid increase in $E(t)$ cannot be overcome by the effects of loop collapse. As a result, $f(t)$ continues to decrease from $t=0$ until it vanishes. At this point, the loop has radiated away all its energy. (Realistically, our calculation breaks down for such small Λ since the radiation is very intense and back-reaction effects will be important.) When Λ is large, the collapse of the loop is the dominant effect on the behaviour of $f(t)$ and hence $f(t)$ grows. This growth can only continue for a while, however, since $E(t)$ is a growing function that blows up at $t = \frac{1}{2} R_0 \pi$. Therefore, $f(t)$ grows for a while, then turns around and starts decreasing. This shows that $f(t)$ always has a maximum value, f_{\max} .

The criterion for black hole formation can now be written as

$$4\pi G \mu > f_{\max}^{-1}, \tag{28}$$

for a given value of Λ . In fig. 3 we display the region of parameter space $(4\pi G \mu, \Lambda)$ where black holes will not form. An important way in which our results differ from the results for gauge strings is that circular gauge string

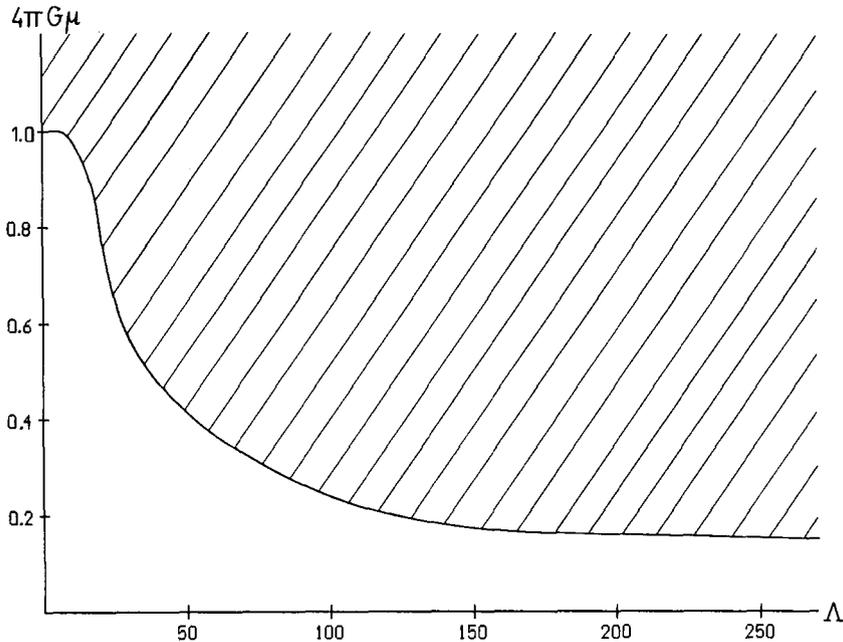


Fig. 3. The region of parameter space (Λ , $4\pi G\mu$) where black holes cannot form is shown as the unhatched region. The hatched region is where it might be possible for black holes to form.

loops of *any* size and tension will collapse to form black holes whereas only *large* loops of relatively massive global string can possibly form black holes. The dependence on the size of the loop is hidden inside the parameter Λ .

Note that, since we have ignored certain factors like the radiation back-reaction, turbulence [3], the gravitational radiation and the universal expansion, we can safely say when black holes will *not* form but we cannot be absolutely sure of when black holes will form. Furthermore, we have only treated the case of a circular loop which is most favoured to collapse to a black hole. If the loop is not circular, black hole formation is even less likely [9].

A specific value of the string tension is relevant if we consider global strings as possible seeds for galaxy formation. Then, $4\pi G\mu \approx 10^{-5}$ and for such strings to form black holes we certainly need $\Lambda > 100$. Circular loops of this size stretch far beyond the current horizon and so we conclude that global strings relevant for galaxy formation will not form black holes.

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