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Hydrodynamics of shallow lakes subject to wind: gyres and geometry

At present complex numerical hydrodynamical models exist, however, easier models should not be forgotten as they really assist in the understanding of the large and fine scale "climatology" of the lakes. Here some of these models used by the authors, and which focus on the dependence of the circulation on geometric factors, are commented on. Further, the coupling effect between the pattern of the wind regime and the lake topography is discussed by using the results of a one-layer model. A guide for writing this model -finite differences (H-M scheme)- is included together with the stability criteria -Fourier method-.

1. Geometry and circulation

Circulation in lakes depends to a large degree on their morphometry. This is the case in standing waves –internal seiches– propagating over the density interface between the upper (mixed) and the bottom layers. Multilayer two dimensional ($2-D$) models reproduce the structure and the frequencies of the internal seiches well [1]. However, as in many elongated lakes internal waves propagate along its main axis, $1-D$ models are still useful tools for experimental scientists [2]. In the case that the lake cannot be considered a two layers system, there is a vertial $1-D$ model which predicts the frequencies of the different vertical modes correctly although their corresponding horizontal structure cannot be obtained [3].

Even easier numerical models are sometimes useful when studying real lakes. That is the case of $0-D$ models which are applied either when the whole lake is homogeneous or when different regions within the lake, and the exchange fluxes between them, can be determined. In fact they simulate baroclinic currents due to the differential cooling between two different basins composing a lake correctly [4] and they are widely used in quality models [5].

However, not only the geometry of the lake but also that of the wind pattern in relation to the lake topography is important. In the next section a classical model to evaluate the coupling effect between both geometries is discussed.

2. Gyres and geometry

Here, a two-dimensional, depth averaged model, simulating the circulation due to the wind in small and medium size lakes is presented. This model is based on the shallow water barotropic equations [6]

$$\frac{\partial u}{\partial t} = -g\frac{\partial \xi}{\partial x} + \frac{\lambda}{h}\sqrt{U^2+V^2}\,U \tag{1}$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial \xi}{\partial y} + \frac{\lambda}{h}\sqrt{U^2+V^2}\,V \tag{2}$$

$$\frac{\partial \xi}{\partial t} = -\frac{\partial (hu)}{\partial x} - \frac{\partial (hv)}{\partial y} \tag{3}$$

where u and v stand for the vertical averaged x and y components of the water velocity, ξ the column water height above $z=0$ level, h the depth of the basin below $z=0$ level, U and V the x and y components of the wind velocity, and, finally, λ is the drag coefficient.

In order to use finite differences, a squared grid covering the studied system has to be defined and then, equations 1–3 can be written as

$$(q_1)^{n+1}_{m\ l} = (q_1)^{n-1}_{m\ l} - \frac{g\tau}{L}[(q_3)^n_{m+1\ l} - (q_3)^n_{m-1\ l}] + \frac{2\tau\lambda}{h_{m\ l}}\sqrt{U^2+V^2}\,U \tag{4}$$

$$(q_2)^{n+1}_{m\ l} = (q_2)^{n-1}_{m\ l} - \frac{g\tau}{L}[(q_3)^n_{m\ l+1} - (q_3)^n_{m\ l-1}] + \frac{2\tau\lambda}{h_{m\ l}}\sqrt{U^2+V^2}\,V \tag{5}$$

$$(q_3)^{n+1}_{m\ l} = (q_3)^{n-1}_{m\ l} - \frac{\tau}{L}[h_{m+1\ l}\,(q_1)^n_{m+1\ l} - h_{m-1\ l}\,(q_1)^n_{m-1\ l} + h_{m\ l+1}\,(q_2)^n_{m\ l+1} - h_{m\ l-1}\,(q_2)^n_{m\ l-1}] \tag{6}$$

where each grid point has been indicated as $(x_m, y_l) = (m\Delta x, l\Delta y)$, with $\Delta x = \Delta y = L$, $m = 1, 2, \ldots, m_{max}$, $l = 1, 2, \ldots, l_{max}$. Time is discretized as $t_n = n\Delta t$ $(\Delta t = \tau, n = 1, \ldots, n_{max})$. The depth data below $z = 0$ level is h_{ml}, the x and y components of the velocity in the space point (x_m, y_l) at time t_n are $(q_1)^n_{m\ l}$ and $(q_2)^n_{m\ l}$, and the fluid height above $z = 0$ level at the same point and time is $(q_3)^n_{m\ l}$. However, if we try to apply these recursive equations as they stand: *a)* For $n = 1$ the initial conditions $(q_i)^1_{m\ l}$, $(q_i)^0_{m\ l}$, $(i = 1, 2, 3)$ are needed. Then we can assume that initially the fluid does not move and so, these values can be set to zero, *i.e.* the wind starts blowing at $n = 1$ (or $t = \tau$). *b)* For $n \geq 1$ the contour conditions $(q_1)^n_{1\ l}$, $(q_1)^n_{m(max)\ l}$, $(q_2)^n_{m\ 1}$, $(q_2)^n_{m\ l(max)}$ are also needed. These values can also be set to zero since we take the normal component of the fluid velocity on the basin borders to be zero. *c)* Note that contour conditions of the fluid height are also needed. In this case, however, we do not know how to fix them so a solution to equations 4–6 cannot be found at all the grid points although we may find the solution at some of them. Following [7], that is using Hansen's scheme, 4–6 can be rewritten as

$$(q_1)^{n+1}_{m+1\ l} = (q_1)^{n-1}_{m+1\ l} - \frac{g\tau}{L}[(q_3)^n_{m+2\ l} - (q_3)^n_{m\ l}] + \frac{2\tau\lambda}{h_{m+1\ l}}\sqrt{U^2 + V^2}\ U \tag{7}$$

$$(q_2)^{n+1}_{m\ l+1} = (q_2)^{n-1}_{m\ l+1} - \frac{g\tau}{L}[(q_3)^n_{m\ l+2} - (q_3)^n_{m\ l}] + \frac{2\tau\lambda}{h_{m\ l+1}}\sqrt{U^2 + V^2}\ V \tag{8}$$

$$(q_3)^{n+2}_{m\ l} = (q_3)^n_{m\ l} - \frac{\tau}{L}[h_{m+1\ l}\ (q_1)^{n+1}_{m+1\ l} - h_{m-1\ l}\ (q_1)^{n+1}_{m-1\ l} + h_{m\ l+1}\ (q_2)^{n+1}_{m\ l+1} - h_{m\ l-1}\ (q_2)^{n+1}_{m\ l-1}] \tag{9}$$

where the indexes are taken to vary as: $m = 2, 4, \ldots, (m_{max} - 3)$, $l = 2, 4, \ldots, (l_{max} - 1)$ in equation 7; $m = 2, 4, \ldots, (m_{max} - 1)$, $l = 2, 4, \ldots, (l_{max} - 3)$ in equation 8; $m = 2, 4, \ldots, (m_{max} - 1)$, $l = 2, 4, \ldots, (l_{max} - 1)$ in equation 9; and $n = 1, 3, 5, \ldots, n_{max}$ in all three equations. m_{max}, l_{max} and n_{max} are taken to be odd numbers. The value of τ has to be given so that the stability of the numerical model is ensured. Using the Fourier method [8] and under some simplifications [7], this will happen if $\tau \leq \frac{L}{2\sqrt{2gH}}$, where H stands for the maximum value of the depth data $h_{m\ l}$.

This model nicely reproduces known circulation patterns in different basin-shapes such as the classical example where there is a bottom channel following the main axis of a lake and the wind blows in the direction of the channel. Then, two gyres with opposite rotation direction appear in such a way that, over the channel, the velocities of both gyres are parallel and they are opposite to the wind direction, while the corresponding returning flow of each gyre, along the shore parallel to the channel, go in the same direction as the wind. More precisely, it can be considered a square lake with sides of 1,100 m and with a bottom channel of 50 m depth crossing the lake through the middle, so that the slope of the bottom from the shores parallel to the channel to the channel is constant. In this case, one hour after a wind of 10 m/s began blowing, water velocity near the shallow shores parallel to the channel was found to be of the order 0.1 m/s, and at the deeper central zone, velocity dropped down to the order of 0.01 m/s. Similar one layer models are also applied to riverine and estuarial systems [9].

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