ELECTRICAL SAFETY

Electric fire hazards at home and in the classroom

According to the National Fire Protection Association [1], in the USA fires in the home kill almost 2900 people every year, or about eight people per day. The majority of heating-related fires are not caused by furnaces, but rather heaters [1]. Sometimes the fuses/RCDs cannot avoid a short-circuit (e.g. because of overcurrent through inadequately rated conductors). Clearly this is a topic of practical importance, with the potential to motivate students. However, physics textbooks rarely deal with electrical security. The few exceptions focus on shock-proof plugs that have a third (grounding) pin, fuses and physiological effects [2–4], but not on fire hazards. Therefore, here we ask the following question: what is the minimum necessary radius of an electric wire to avoid fire hazards?

This topic is not as straightforward as it might seem. Indeed, the electric power is $P = VI$, with V the grid voltage and I the intensity, so Ohm's law ($V = IR$, with R the resistance) yields $P = V^2/R$. However, this does not give information about the conditions under which the power P of an electric device is too high to heat the wires and so could possibly cause a fire. In fact, as we shall see explicitly below, $P = VI$ is the total power supplied by the grid but not the power dissipated by the electric wires, which is the relevant quantity to determine whether the wires will heat up and eventually cause a fire. To solve the problem posed we therefore have to distinguish carefully between the power supplied by the grid and that lost by the electric device and dissipated as heat in the wires.

Theory

Consider an electrical device (e.g. a heater) connected to a plug through a metal wire (i.e. an extension cord). As usual, we are dealing in fact with two wires, one carrying the current to the device and the second returning it to the grid; for an alternating current (AC), both wires exchange roles periodically. Each wire is surrounded by a plastic cover and usually these are joined together so that in practice the wires are parallel to each other and separated by plastic material of about one millimeter or less. We need to determine the minimum radius of the metal wires so they will not heat up and melt their plastic covers. If this happens, both metal wires could come into contact with each other and eventually cause a fire. The total power supplied by the electrical grid is

$$P_{grid} = P_{device} + P_{wire} \,, \tag{1}$$

where P_{device} is the power of the device (e.g. 1000 W for a typical home heater) and P_{wire} is the total power dissipated at the wires. Assume that $P_{\mathrm{device}} >> P_{\mathrm{wire}}$,

$$P_{grid} \approx P_{device} \,. \tag{2}$$

Then we can rewrite Eq. (2) as

$$VI \approx P_{device} \,. \tag{3}$$

We should recall that if we are dealing with AC, then V and I are root-mean-square values and we should consider the average powers [5]. The power dissipated at the wires as heat (by Joule's effect) is

$$P_{wire} = R_{wire} I^2 \approx R_{wire} \left(\frac{P_{device}}{V} \right)^2 , \tag{4}$$

where R_{wire} is the resistance of the wires. In general, the time evolution of the wire temperature T can be obtained by integrating the following differential equation [3]

$$C \frac{dT}{dt} = R_{wire} \left(\frac{P_{device}}{V} \right)^2 - Aq(T-T_0) - Ae\sigma(T^4 - T_0^4) - \frac{A\kappa}{r}(T-T_0) \,, \tag{5}$$

where the first term on the right-hand side of the equation is the electrical power (which tends to increase the wire temperature) and the other terms are the heat lost per unit time due to convection, radiation and conduction, respectively. C is the thermal capacity of the wire, A its lateral area ($A = 2\pi rl$, with r the radius and l the length), q the convection coefficient [3], e the emissivity, σ is Stefan's constant and κ is the thermal conductivity [3]. For our purposes, we can simplify this model by noting that the thermal conductivity of air is very small, so it is a very good insulator [5] and the last term (heat conduction) can be neglected. Similarly, the emissivity of metals (and their plastic covers) is very small [5], so the second term on the right-hand side (heat radiation) can also be neglected. In other words, the main heat transport mechanism is convection [5] and

$$C \frac{dT}{dt} \approx R_{wire} \left(\frac{P_{device}}{V} \right)^2 - Aq(T-T_0) \,. \tag{6}$$

The wires will heat up if the electrical power, which tends to increase the temperature (the first term on the right), exceeds the power dissipated into the environment (the last term). Therefore, the condition for the wires to heat up is

$$R_{wire} \left(\frac{P_{device}}{V} \right)^2 > \kappa(T-T_0) \,, \tag{7}$$

where $\kappa = Aq$. Thus the wires will heat up if their resistance is larger than a threshold value, which depends on the electrical and thermal properties of the system, namely

$$R_{wire} > \kappa(T-T_0) \left(\frac{V}{P_{device}} \right)^2 . \tag{8}$$

Taking into account that $R_{\mathrm{wire}} = \rho l/S$, where ρ is the resistivity of the metal, l the total length of the wires and S their cross-section, we finally find the minimum cross-sectional area of the metal wires for them not to heat up, namely

$$S_{\min} = \frac{\rho l}{\kappa (T - T_0)} \left(\frac{P_{device}}{V} \right)^2, \qquad (9)$$

which increases with the power of the device, as we could have expected intuitively. This is why an extension cord of thin wire that works properly when we use it to plug a low-power device (e.g. a 50 W light bulb) may be very dangerous, and even cause a fire, if we use it to plug a high-power device (e.g. a 1000 W heater) instead. Only sufficiently thick wires should be used to plug high-power electrical devices.

Consider a practical situation, as follows. We plug an electrical device, which is initially at room temperature ($T = T_0$). Obviously equation (7) holds and thus the wire begins to heat up. Some time later, T will have increased and, if equation (7) no longer holds, the wire will stop heating up. This will in general depend on the variation in the resistance $R_{wire} = \rho l/S$ with temperature. Note that ρ usually increases with temperature, and this makes it more difficult for equation (7) to break down as T rises (this is sometimes called the thermal runaway effect). A fire may be caused if condition (7) holds up to the point at which both wires come into physical contact with each other, i.e. at the melting temperature of the plastic cover. Thus, the temperature T in equation (9) corresponds to the melting temperature for the plastic considered, usually about $T = 100$ °C. Alternatively, we could use the plastic softening temperature, which is somewhat lower, because this may be enough for both metal wires to come into contact with each other, but this distinction is not necessary to explain the main physical points of heating fire hazards.

Problem solving

Students substantially increase their understanding of physics if they apply theoretical concepts and models to specific problems [6,7]. Problem solving also helps them to remember important conclusions, such as those reached in the paragraph below equation (9). Accordingly, we propose a simple problem that we have found very useful for our first-year undergraduate students.

Problem. We want to use an extension cord (length $l = 5$ m, cooling coefficient $\kappa = 1.72 \cdot 10^{-3}$ W/°C) of copper wires ($\rho = 1.68 \cdot 10^{-8}$ Ω m) to plug a light bulb ($P_{device} = 50$ W) to the grid ($V = 220$ V) at a room temperature of $T_0 = 20$ °C. What is the minimum radius of the cross-sectional area of the wires required for them not to heat up and eventually cause a fire? How would the answer change if we plugged an electric heater ($P_{device} = 1000$ W) rather than a light bulb? The melting temperature of the plastic that surrounds the metal wires is $T = 100$ °C.

Solution. Putting these values into equation (9), we find that the minimum wire radius is $r_{min} = \sqrt{(S_{min}/\pi)} = 0.1$ mm for the light bulb. To plug the heater safely the minimum wire radius is much larger, $r_{min} = 2$ mm. Therefore, when plugging electrical devices to extension cords, paying attention to the wire cross-section is of the utmost importance to avoid fire hazards.

Conclusions

Fire hazards in domestic electricity are of substantial practical importance and can easily gain the attention and interest of secondary-school and first-year undergraduate science and engineering students (we suggest that for secondary-school students the differential equations (5)–(6) may be omitted). This topic serves to illustrate the usefulness of basic physics principles very well. Moreover, it provides useful advice in avoiding fires at home when using extension cords that are too narrow to plug heaters or other high-power devices.

References

[1] Detailed information is available at the website of the National Fire Protection Association www.nfpa.org/
[2] Cutnell J D, Johnson K W 1997 *Physics* (New York: Wiley)
[3] Kane J W, Sternheim M M 1984 *Physics* 2nd edn (New York: Wiley)
[4] Serway R A 1992 *Physics for Scientists and Engineers* (Philadelphia, PA: Saunders)
[5] Tipler P A 1991 *Physics for Scientists and Engineers* extended edn, chapter 28 (New York: Worth)
[6] Plotzner R 1994 *The Integrative Use of Qualitative and Quantitative Knowledge in Physics Problem Solving* (Frankfurt: Peter Lang)
[7] Bolton J 1997 Developing student's physics problem-solving skills *Phys. Educ.* **32** 176

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